arXiv:physics/9810028v1 [physics.atom-ph] 15 Oct 1998

Matter wave interference using two-level atoms and resonant optical fields

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(February 2, 2008)

A theory of matter wave interference is developed in which resonant, standing wave optical fields interact with an ensemble of two-level atoms. If effects related to the recoil the atoms undergo on absorbing or emitting radiation are neglected, the *total* atomic density is spatially uniform. However, when recoil effects are included, spatial modulation of the atomic density can occur for times that are greater than or comparable with the inverse recoil frequency, ω_q^{-1}. In this regime, the atoms exhibit matter-wave interference that can be used as the basis of a matter wave atom interferometer. Two specific atom field geometries are considered. In the first, atoms characterized by a homogeneous velocity distribution are subjected to a single radiation pulse. The pulse excites the atoms which then decay back to the lower state. The spatial modulation of the total atomic density is calculated as a function of t, where t is the time following the pulse. In contrast to the normal Talbot effect, the spatially modulated density is *not* a periodic function of t, owing to spontaneous emission; however, after a sufficiently long time, the contribution from spontaneous processes no longer plays a role and the Talbot periodicity is restored. In the second atom-field geometry, there are two pulses separated by an interval T. The atomic velocity distribution in this case is assumed to be inhomogeneously broadened. Owing to the inhomogeneous broadening, one finds a nonvanishing spatial modulation of the density only at specific "echo times" following the second pulse. In contrast to the normal Talbot-Lau effect, the spatially modulated density is *not* a periodic function of T, owing to spontaneous emission; however, for sufficiently long time, the contribution from spontaneous processes no longer plays a role and the Talbot periodicity is restored. The structure of the spatially modulated density in the vicinity of the echo times is studied, and is found to mirror the atomic density following the first pulse. With a suitable choice of observation time and field strengths, the spatially modulated atomic density serves as an indirect probe of the distribution of spontaneously emitted radiation.

03.75.Be, 39.20+q, 32.80.Lg

I. INTRODUCTION

Atom interferometry has emerged as an important new research area in the past fifteen years [1]. Thermal atoms offer unique properties as the working element of an interferometer, owing to their small de Broglie wavelength. Atom interferometry has already led to impressive results in precision measurements of $\hbar$ [2], atomic polarizability [3], rotation rates [4,5] and the acceleration of gravity [6]. Applications to nanolithography have also been proposed [7]. Despite the advances in atom interferometry, there remains some controversy as to what constitutes an atom interferometer. We have discussed previously [8] the distinction between classical and quantum scattering for atom interferometers in which microfabricated gratings are used as the "optical" elements in the interferometer. In this paper, we extend this discussion to atom interferometers that employ optical fields to modify the atomic state wave function.

In our effort to classify atom interferometers, it will prove useful to summarize the conclusions we reached in analyzing the scattering of atoms by microfabricated gratings [8]. As a model system, we consider the scattering of an atomic beam by two microfabricated gratings separated by a distance L. The internal structure of the atoms is irrelevant here, since the atoms are in their ground state and the scattering by the gratings is assumed to be the same for any and all ground state sublevels - there is no internal state coherence. For an angular divergence of the atomic beam, θ_b, satisfying $\theta_b \gg d/L$, where d is the grating period, the atomic density is uniform in the region between the gratings, but can exhibit "fringes" beyond the second grating. In particular, at a distance $2L$ from the first grating, fringes having period d are produced. As such, this device has the appearance of an interferometer. Moreover, it can be used to measure inertial effects [9], since the fringes are displaced if the atoms accelerate between the interaction zones. Further evidence that this device constitutes an interferometer is provided by a theoretical description of the scattering in which the microfabricated gratings create coherence between transverse momentum components of the incoming atomic beam, taken as a plane wave. Despite all these features, it is incorrect to conclude that such a system legitimately qualifies as an atom interferometer.

To clarify matters, it is best to back up a bit and consider the scattering of an atomic beam by a single grating. At a distance L from the grating, the nature of the scattering is determined by the so called Talbot length [10], defined as

$$L_T = 2d^2/\lambda_{dB},$$

where $\lambda_{dB} = h/mu_0$ is the atomic de Broglie wavelength (m is the atomic mass and u_0 is the longitudinal atomic speed). For $L \ll L_T$, diffraction effects are unimportant and one is in the classical scattering limit. For a highly collimated beam ($\theta_b \ll d/L$), the pattern produced on a screen at a distance $L \ll L_T$ from the microfabricated grating is simply the *shadow* of the grating. This is clearly a classical effect.

Of course, the classical scattering limit can be obtained from a calculation in which the center-of-mass motion is quantized. If such a fully quantum calculation is carried out, one finds that, immediately following the grating, the atomic state wave function contains coherence between different transverse momentum components. The *atomic state amplitude grating* or matter grating created by the microfabricated grating necessarily implies the existence of such coherence. However, for $L \ll L_T$, this coherence is *not* evidence for quantum scattering since it corresponds simply to the classical shadow effect. The relative phase associated with the coherence associated with two transverse momentum components, p and $p + \hbar q$, with $\hbar q \equiv 2\pi\hbar/d$, can be written as

$$\phi = q\frac{p}{m}t + \omega_q t,$$

where $t = L/u_0$ and $\omega_q = \hbar q^2/2m$ is a recoil frequency. The first term is a Doppler shift, independent $\hbar$ - it has a classical origin. The second term is a recoil shift which vanishes in the limit $\hbar \sim 0$. It is a purely quantum term. Thus, for $\omega_q t \ll 1$, which is equivalent to the condition $L \ll L_T$, the scattering is classical and the coherence between transverse momentum components corresponds to a classical Doppler shift. It should now be clear that the fringes produced in the two-grating interferometer have nothing to do with quantization of the atomic motion when $L \ll L_T$. They result simply from a shadow effect and constitute a moiré pattern. For grating separations $L \gtrsim L_T$, quantum diffraction effects begin to play a role; however, the conditions for fringe production are still determined by classical considerations related to cancellation of a Doppler phase.

We return now to atom interferometers which use optical fields to create atomic coherence between internal states, external states, or both. Depending on which coherence is created and probed, one can classify such atom interferometers into three general categories. Classical atom interferometers (CAI) in no way rely on quantization of the atoms' center-of-mass motion for their operation. The CAI operate in the regimes $\omega_q t \ll 1$ or $L \ll L_T$, for which quantum scattering effects can be ignored. If one restricts the use of the term "atom interferometer" to those devices which depend on quantization of the atoms' center-of-mass motion for their operation, the CAI would not qualify. Examples of CAI include those based on radio-frequency [11] or optical [12–14] Ramsey fringes.

Matter wave atom interferometers operate only in the regime $\omega_q t \gtrsim 1$ or $L \gtrsim L_T$. We distinguish matter wave atom interferometers in which the *total* atomic density is monitored from those in which an internal state coherence or population is probed. The former we refer to simply as matter wave atom interferometers (MWAI) and the latter as internal state matter wave atom interferometers (ISMWAI). An important feature which distinguishes MWAI from ISMWAI is the dependence of the observed signal on the parameter $\omega_q t$ or L/L_T. The ISMWAI signal does not vanish in the limit $\omega_q t \ll 1$ or $L \ll L_T$, but *does* vanish in this limit for MWAI. In other words, MWAI depend critically on the quantization of the atoms' center of mass motion for their operation, whereas ISMWAI can operate in the classical or quantum scattering limits. Although matter wave effects can modify the signals in ISMWAI, many features of the signals are determined by classical considerations, as in CAI. Examples of ISMWAI interferometers include those which explore recoil splitting of optical Ramsey fringes in the frequency domain [15–18,2,19], and time-domain [6,20,21].

Matter wave atom interferometers can be constructed using off-resonant standing wave optical fields. Such fields act as phase gratings for the matter waves. Atoms enter a field interaction region in their ground state and leave the region with their spatial density unaffected. On the other hand, the *phase* of the ground state wave function is spatially modulated, corresponding to a coherence between momentum states differing by $n\hbar q$ (n is an integer and $4\pi/q$ is the period of the standing wave field). As the matter wave evolves freely following the field interaction region, the phase modulated wave function is transformed into a spatially modulated density, which can be probed using various techniques. Atom interferometers can be constructed using two or more field interaction zones [22–24]. In the transient echo experiment of Cahn *et al.* [23], the vanishing of spatial modulation at the echo times $t = 2T$ and $t = 3T$ (but not in the immediate vicinity of the echo times) is consistent with MWAI theory [25]. Cahn *et al.* also found that the spatial modulation vanished in the classical scattering region, $\omega_q T \ll 1$, as it must for any MWAI.

It is also possible to create MWAI by resonantly driving one-photon transitions between electronic states [25] or two-photon Raman transitions between ground state sublevels using optical fields. The lifetime of the electronic or

ground state coherence must be greater than ω_q^{-1}, a condition that restricts one to intercombination lines in the case of electronic state coherence. It is possible to show [25] that the sum of the spatially modulated atomic gratings created by the fields in each of the states involved in the transition (that is, the total atomic density) vanishes in the classical scattering limit, but is nonvanishing in the quantum scattering domain, $\omega_q t \gtrsim 1$. This effect has not yet been observed. If one were to probe the total atomic density rather than some internal state density in the experiments of [16,17,19], the sought after effect should be observable.

In this paper, we describe a new type of MWAI [26] that also uses resonant rather than off-resonant fields. Two-level atoms pass through a standing wave, optical field that resonantly drives a *closed* two-level transition in the atoms. In contrast to the scheme described above, it is now assumed that the excited state lifetime is much less than ω_q^{-1}. As a result of the atom-field interaction, an excited state matter grating in the excited state is created, as is a "hole grating" in the ground state. If one neglects all effects related to quantization of the atoms' center-of-mass motion, it is easy to show that, after the excited state decay back to the ground state, the ground state density is uniform. In some sense, the excited state grating fills the "hole" in the ground state density that had been created by the field. When the recoil associated with stimulated [27] and spontaneous [28,29] processes are included, however, new features appear in the ground state density matrix. It is not possible to interpret the interaction in simple terms as was the case for a phase grating, but the net result is that recoil leads to a nonvanishing spatially modulated atomic density for $\omega_q t \gtrsim 1$ or $L \gtrsim L_T$. In analyzing the signal, it will prove useful to separate the contributions from spontaneous and stimulated processes. Immediately following the excited state decay, these two contributions cancel one another, but as time progresses, the contribution from spontaneous processes disappears, leaving a net modulated ground state density. When two field interaction zones are used, echo-like phenomena can occur. It will be seen that *both* stimulated and spontaneous processes contribute to the echo signals, even for pulse separations $T \gg \omega_q^{-1}$.

The paper is arranged as follows. The change in the atomic density matrix following the interaction of atoms with a standing wave, optical field is calculated in Sec. II. In Sec. III we consider atom interference using a single atom-field interaction zone and a homogeneous distribution of atomic velocities. It is seen that focussing of the atoms, similar to that found with phase gratings, can also occur using resonant fields. The conditions under which the spatially modulated atomic density becomes a periodic function of t (Talbot effect) are established. The atomic density following two atom-field interaction zones is calculated for an inhomogeneously broadened transition (Talbot-Lau regime) in Sec. IV. The results are summarized in Sec. V.

II. BASIC EQUATIONS

Atom interferometers can operate in the spatial or time domain. In the spatial domain an atomic beam traverses one or more field regions. In the time domain, a vapor of cold atoms (or condensate) is subjected to one or more radiation pulses. The spatial domain interferometer can be analyzed in the time domain if calculations are carried out in the atomic rest frame. Consequently, we restrict our calculations to the time domain.

Two-level (upper state $|e\rangle$ and ground state $|g\rangle$) atoms are subjected to two radiation pulses separated by a time interval T. *Each* radiation pulse consists of two traveling wave components having propagation vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, $|\mathbf{k}_1| = |\mathbf{k}_2| = k = \Omega/c$, and Ω is the frequency of each field. The total electric field can be written as

$$E(\mathbf{r},t) = \hat{\mathbf{e}} e^{-i\Omega t + i\mathbf{Q}\cdot\mathbf{r}} \cos(\mathbf{q}\cdot\mathbf{r}/2)\left[E_1 g_1(t) + E_2 g_2(t-T)\right] + c.c., \tag{1}$$

where $\mathbf{Q} = (\mathbf{k}_1 + \mathbf{k}_2)/2$, $\mathbf{q} = (\mathbf{k}_1 - \mathbf{k}_2)$, $\hat{\mathbf{e}}$ is a polarization vector, E_j is the amplitude of pulse j ($j = 1, 2$), and $g_j(t)$ is a smooth pulse envelope function having width τ, centered at $t = T_j$, with $T_1 = 0$ and $T_2 = T$. We assume that the pulse separation T, pulse duration τ, atom-field detuning $\Delta = \Omega - \omega$ (ω is the atomic transition frequency), upper state decay rate Γ, recoil frequency

$$\omega_q = \hbar q^2/2m, \tag{2}$$

and Doppler shift $\mathbf{k}_i \cdot \mathbf{v}$ ($\mathbf{v}$ is an atomic velocity) satisfy the inequalities

$$\Gamma T \gg 1, \tag{3a}$$
$$\Gamma\tau \ll 1, \tag{3b}$$
$$\Delta\tau \ll 1, \tag{3c}$$
$$\mathbf{k}_i \cdot \mathbf{v} \ll \Gamma, \tag{3d}$$
$$\omega_q T \geq 1. \tag{3e}$$

Inequality (3a) implies that any excited state population created by the fields decays to the ground state in a time that is short compared with the time scale of the experiment. Inequalities (3b, 3c) imply that spontaneous decay

and atom-field detuning can be neglected during the radiation pulses, while inequality (3d) (atoms cooled below the Doppler limit of laser cooling) guarantees that there is negligible Doppler dephasing for times of order of the excited state lifetime. Finally condition (3e) states that we are in the quantum scattering limit.

Conditions (3) allow one to map out the time development of the density matrix resulting from each radiation pulse in three stages:

1. an impulsive change in the density matrix produced by the atom-field interaction
2. spontaneous decay of the excited state
3. free evolution of the density matrix.

Depending on the specific application, we calculate the atomic density matrix following the first or second pulse. Owing to inequalities (3), we can define times before ($T_j^{(-)}$) and following ($T_j^{(+)}$) pulse j such that changes in the atomic density resulting from spontaneous decay and atom-field detunings can be neglected in the time interval $T_j^{(+)} - T_j^{(-)}$.

A. Stage 1

During pulse j, the density matrix, in an interaction representation, evolves according to

$$i\hbar\dot{\rho} = [V, \rho]\,, \tag{4}$$

$$V = 2\hbar\chi_j g_j\left(t - T_j\right)\cos\left(\mathbf{q}\cdot\mathbf{r}/2\right)\left[\cos\left(\mathbf{Q}\cdot\mathbf{r}\right)\sigma_x - \sin\left(\mathbf{Q}\cdot\mathbf{r}\right)\sigma_y\right], \tag{5}$$

where $\chi_j = -\mu E_j/2\hbar$ is a Rabi frequency, μ is a dipole moment operator matrix element, σ_x and σ_y are Pauli matrices, and

$$|e\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ |g\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{6}$$

It has been assumed that, during each pulse, any effects arising from atomic motion can be neglected [inequalities (3b, 3d)], which is the reason the kinetic energy term has been omitted from the Hamiltonian in Eq. (4) (Raman-Nath approximation). Before the first pulse acts, it is assumed that all atoms are in their ground state. Owing to inequality (3a), all population is returned to the ground state before the action of the second pulse at $t = T_2$. Thus we need only calculate the change in the ground and excited state density matrices produced by pulse j, starting from a density matrix

$$\rho\left(\mathbf{r}, \mathbf{r}'; T_j^{(-)}\right) = \begin{pmatrix} 0 & 0 \\ \rho_{gg}\left(\mathbf{r}, \mathbf{r}'; T_j^{(-)}\right) & 0 \end{pmatrix}, \tag{7}$$

in which all atoms are in their ground state. One can integrate Eq. (4) to obtain the density matrix immediately following the pulse at time $T_j^{(+)}$,

$$\rho\left(\mathbf{r}, \mathbf{r}'; T_j^{(+)}\right) = \eta_j\left(\mathbf{r}\right)\rho\left(\mathbf{r}, \mathbf{r}'; T_j^{(-)}\right)\eta_j^{\dagger}\left(\mathbf{r}'\right), \tag{8a}$$

$$\eta_j\left(\mathbf{r}\right) = \cos\left[{}^1\!/_2\theta_j\cos\left(\mathbf{q}\cdot\mathbf{r}/2\right)\right] - i\sin\left[{}^1\!/_2\theta_j\cos\left(\mathbf{q}\cdot\mathbf{r}/2\right)\right]\left[\cos\left(\mathbf{Q}\cdot\mathbf{r}\right)\sigma_x - \sin\left(\mathbf{Q}\cdot\mathbf{r}\right)\sigma_y\right], \tag{8b}$$

where

$$\theta_j = 4\chi_j\int_{-\infty}^{\infty} dt g_j\left(t\right) \tag{9}$$

is a pulse area.

It is convenient to use the Wigner representation for the density matrix,

$$\rho\left(\mathbf{r}, \mathbf{p}, t\right) = \int \frac{d\hat{\mathbf{r}}}{\left(2\pi\hbar\right)^3}\exp\left(-i\mathbf{p}\cdot\hat{\mathbf{r}}/\hbar\right)\rho\left(\mathbf{r} + \hat{\mathbf{r}}/\mathbf{2}, \mathbf{r} - \hat{\mathbf{r}}/\mathbf{2}, t\right),$$

and expand the populations as

$$\rho_{nn}\left(\mathbf{r},\mathbf{p},t\right)=\sum_{s}\rho_{nn}\left(s,\mathbf{p},t\right)\exp\left[is\mathbf{q}\cdot\mathbf{r}\right]. \tag{10}$$

Using Eqs. (8)-(10) and expanding the $\sin\left[{}^{1}/_{2}\theta_{j}\cos\left(\mathbf{q}\cdot\mathbf{r}/2\right)\right]$ and $\cos\left[{}^{1}/_{2}\theta_{j}\cos\left(\mathbf{q}\cdot\mathbf{r}/2\right)\right]$ functions in terms of Bessel functions, one obtains for the Fourier coefficients

$$\rho_{gg}\left(s,\mathbf{p},T_{j}^{(+)}\right)=\sum_{\ell,s'}\left(-1\right)^{s'}J_{2\ell}\left(\theta_{j}/2\right)J_{2(\ell-s')}\left(\theta_{j}/2\right)\rho_{gg}\left[s-s',\mathbf{p}-\hbar\mathbf{q}\left(\ell-s'/2\right),T_{j}^{(-)}\right], \tag{11a}$$

$$\rho_{ee}\left(s,\mathbf{p},T_{j}^{(+)}\right)=\sum_{\ell,s'}\left(-1\right)^{s'}J_{2\ell+1}\left(\theta_{j}/2\right)J_{2(\ell-s')+1}\left(\theta_{j}/2\right)\rho_{gg}\left\{s-s',\mathbf{p}-\hbar\left[\mathbf{q}\left(\ell-\left(s'-1\right)/2\right)+\mathbf{Q}\right],T_{j}^{(-)}\right\}, \tag{11b}$$

where $J_{s}(x)$ is a Bessel function of order s.

B. Stages 2 and 3

In the next stages of the calculation, the excited state decays to the ground state and the ground state density matrix evolves freely following the decay. The Wigner function associated with the excited state population obeys the equation of motion

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\rho_{ee}\left(\mathbf{r},\mathbf{p},t\right)=-\Gamma\rho_{ee}\left(\mathbf{r},\mathbf{p},t\right) \tag{12}$$

It then follows from Eq. (12) that the s-order Fourier component evolves as

$$\rho_{ee}\left(s,\mathbf{p},t\right)=\exp\left[-\left(\Gamma+is\mathbf{q}\cdot\mathbf{p}/m\right)\left(t-T_{j}\right)\right]\rho_{ee}\left(s,\mathbf{p},T_{j}^{(+)}\right) \tag{13}$$

The excited state repopulates the ground state. The ground state Wigner function is governed by the equation

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\rho_{gg}\left(\mathbf{r},\mathbf{p},t\right)=\Gamma\int d\mathbf{n}_{r}N\left(\mathbf{n}_{r}\right)\rho_{ee}\left(\mathbf{r},\mathbf{p}+\hbar\mathbf{k}_{r},t\right), \tag{14}$$

from which it follows that the ground state Fourier coefficients evolve as

$$\left(\frac{\partial}{\partial t}+is\mathbf{q}\cdot\mathbf{p}/m\right)\rho_{gg}\left(s,\mathbf{p},t\right)=\Gamma\int d\mathbf{n}_{r}N\left(\mathbf{n}_{r}\right)\rho_{ee}\left(s,\mathbf{p}+\hbar\mathbf{k}_{r},t\right) \tag{15}$$

In these equations, $\mathbf{k}_{r}$ is a spontaneous photon wave vector, $\mathbf{n}_{r}=\mathbf{k}_{r}/k_{r}$, and $N\left(\mathbf{n}_{r}\right)$ is the normalized probability density for the radiation of a photon in the direction $\mathbf{n}_{r}$.

The solution of Eq. (14) involves both a homogeneous and particular solution which we write as

$$\rho_{gg}\left(s,\mathbf{p},t\right)=\rho_{gg}^{(S)}\left(s,\mathbf{p},t\right)+\rho_{gg}^{(D)}\left(s,\mathbf{p},t\right). \tag{16}$$

The homogeneous part,

$$\rho_{gg}^{(S)}\left(s,\mathbf{p},t\right)=\exp\left[-is\mathbf{q}\cdot\mathbf{p}\left(t-T_{j}\right)/m\right]\rho_{gg}\left(s,\mathbf{p},T_{j}^{(+)}\right), \tag{17}$$

represents the evolution of the ground state in the absence of decay. The S superscript indicates that this part is associated purely with stimulated processes. The particular solution, $\rho_{gg}^{(D)}\left(s,\mathbf{p},t\right)$, represents the contribution to the ground state density matrix resulting from excitation by the pulse and subsequent decay of the excited state (hence, the superscript D for decay). One finds that at times $t-T_{j}\gg\Gamma^{-1}$ the particular solution is given by

$$\rho_{gg}^{(D)}\left(s,\mathbf{p},t\right)=\exp\left[-is\mathbf{q}\cdot\mathbf{p}\left(t-T_{j}\right)/m\right]\int d\mathbf{n}_{r}N\left(\mathbf{n}_{r}\right)\left(1+is\omega_{d}n_{q}/\Gamma\right)^{-1}\rho_{ee}\left(s,\mathbf{p}+\hbar\mathbf{k}_{r},T_{j}^{(+)}\right), \tag{18}$$

where

$$\omega_d = \hbar q k_r/m \tag{19}$$

and $n_q = \mathbf{n}_r\cdot\mathbf{q}/q$. The quantity $s\omega_d$ is a recoil frequency associated with the spontaneous decay of the sth excited state Fourier component. Alternatively, $s\omega_d$ can be viewed as a Doppler shift of the spontaneously emitted photon that is dependent on the momentum kick $s\hbar q$ which the atom acquires in the excitation process.

Piecing together Eqs. (18), (17), (11), one finds that the ground state density matrix for $t - T_j \gg \Gamma^{-1}$ is given by

$$\begin{aligned}\rho_{gg}(s,\mathbf{p},t) = &\sum_{\ell,s'}(-1)^{s'}\exp\left[-is\mathbf{q}\cdot\mathbf{p}\,(t-T_j)/m\right]\Big\{J_{2\ell}(\theta_j/2)\,J_{2(\ell-s')}(\theta_j/2)\,\rho_{gg}\left[s-s',\mathbf{p}-\hbar\mathbf{q}\,(\ell-s'/2),T_j^{(-)}\right]\\ &+J_{2\ell+1}(\theta_j/2)\,J_{2(\ell-s')+1}(\theta_j/2)\int d\mathbf{n}_r N(\mathbf{n}_r)\,(1+is\omega_d n_q/\Gamma)^{-1}\\ &\times\rho_{gg}[s-s',\mathbf{p}-\hbar\,(\mathbf{q}\,(\ell-s'/2)+\mathbf{Q}-\mathbf{k}_r),T_j^{(-)}]\Big\}\end{aligned} \tag{20}$$

This is the building-block solution which can be used to analyze several possible experimental schemes.

For $\omega_d^{-1},\omega_q^{-1} \gg t - T_j \gg \Gamma^{-1}$, recoil effects can be neglected and the terms involving recoil momenta in the arguments of the density matrix elements can be dropped. In that limit the sum over ℓ of the Bessel functions gives $\delta_{s',0}$, and Eq. (20) reduces to $\rho_{gg}(s,\mathbf{p},t) = \exp\left[-is\mathbf{q}\cdot\mathbf{p}\,(t-T_j)/m\right]\rho_{gg}(s,\mathbf{p},T_j^{(-)})$, which implies that

$$\rho_{gg}(\mathbf{r},\mathbf{p},t) = \rho_{gg}\left(\mathbf{r}-\frac{\mathbf{p}}{m}(t-T_j),\mathbf{p},T_j^{(-)}\right). \tag{21}$$

As expected, the density matrix simply undergoes a classical translation if recoil is neglected.

III. ONE FIELD-INTERACTION ZONE

When an electromagnetic wave passes through an amplitude or phase grating, the diffraction pattern as a function of the distance from the grating is a periodic function of the Talbot length, or, equivalently, of $\omega_q t$. Similar effects occur for matter waves and have been observed experimentally by Chapman et al [30] and Nowak et al [31]. In these experiments, one sent ground state atoms through a microfabricated grating. When atoms are excited by resonant, optical fields, the diffraction pattern is no longer strictly periodic owing to spontaneous decay. In this section we calculate the atomic density following the interaction of a highly collimated atomic beam ($qu\theta_b t \ll 1$) or a condensate ($qut \ll 1$) with a resonant optical field. In the atomic rest frame, the field appears as a pulse centered at $T_1 = 0$. The initial Wigner distribution is taken as $\rho_{gg}\left(\mathbf{r},\mathbf{p},T_1^{(-)}\right) = W(\mathbf{p})$ [32], corresponding to Fourier components

$$\rho_{gg}\left(s,\mathbf{p},T_1^{(-)}\right) = \delta_{s,0}W(\mathbf{p}). \tag{22}$$

In this equation, $W(\mathbf{p})$ is the momentum distribution of the atoms.

It is convenient to orient the $x,\ y,\ z$ axes along the mutually orthogonal vectors $\mathbf{q},\ \mathbf{Q}$, and $\mathbf{k}_1\times\mathbf{k}_2$. It then follows from Eqs. (20), along with the summation identities

$$\begin{aligned}S_\nu^{(e)}(a) &\equiv \sum_\ell J_{2\ell}(a)\,J_{2(\ell-\nu)}(a)\exp(-i\ell\alpha)\\ &= \frac{1}{2}\exp^{-i\nu\alpha/2}\left\{J_{2\nu}\left[2a\cos(\alpha/4)\right]+(-1)^\nu J_{2\nu}\left[2a\sin(\alpha/4)\right]\right\},\end{aligned} \tag{23a}$$

$$\begin{aligned}S_\nu^{(o)}(a) &\equiv \sum_\ell J_{2\ell+1}(a)\,J_{2(\ell-\nu)+1}(a)\exp(-i\ell\alpha)\\ &= -\frac{1}{2}\exp^{-i(\nu-1)\alpha/2}\left\{J_{2\nu}\left[2a\cos(\alpha/4)\right]-(-1)^\nu J_{2\nu}\left[2a\sin(\alpha/4)\right]\right\},\end{aligned} \tag{23b}$$

that the Fourier component at time t,

$$\rho_{gg}(s,t) = \int d\mathbf{p}\rho_{gg}(s,\mathbf{p},t), \tag{24}$$

is given by

$$\rho_{gg}(s,t) = {}^{1}/_{2}\left\langle \exp\left(-is\mathbf{q}\cdot\mathbf{p}t/m\right)\right\rangle \left\{J_{2s}\left[\theta_1 \sin\left(\phi_{T_S}(st)/2\right)\right]\left[1+C\left(\phi_{T_D}(st), s\omega_q/\Gamma\right)\right]\right.$$
$$\left.+(-1)^s J_{2s}\left[\theta_1\cos\left(\phi_{T_S}(st)/2\right)\right]\left[1-C\left(\phi_{T_D}(st), s\omega_d/\Gamma\right)\right]\right\}, \tag{25}$$

where $\langle\ldots\rangle$ represents an average over atomic momenta,

$$\phi_{T_S}(t) = \omega_q t \tag{26}$$

is the Talbot phase associated with stimulated processes,

$$\phi_{T_D}(t) = \omega_d t \tag{27}$$

is the Talbot phase associated with spontaneous processes, and

$$C(\alpha,\beta) = \int d\mathbf{n} N(\mathbf{n}) \exp(i\alpha n_x)\left(1+i\beta n_x\right)^{-1}. \tag{28}$$

Terms involving $C(\alpha,\beta)$ in Eq. (25) are connected with spontaneous processes while the remaining terms arise from stimulated processes.

It follows from Eq. (25) that the spatially homogeneous part of the atomic density is unchanged,

$$\rho_{gg}(0,t) = 1, \tag{29}$$

consistent with probability conservation in the closed two-level system. We are interested primarily in the time dependence of the Fourier components having $s \neq 0$, since these components determine the spatial modulation of the atomic density. The maximum value of s entering the summation in Eq. (10) is of order $\max\{1,\theta_1\}$. It is assumed in this section that the Doppler broadening is small,

$$\max\{1,\theta_1\}\,\mathbf{q}\cdot\mathbf{p}t/m \ll 1, \tag{30}$$

which allows one to set $\langle\exp(-is\mathbf{q}\cdot\mathbf{p}t/m)\rangle$ equal to unity in Eq. (25). Note that, even without this factor, the general expression (25) is *not* a periodic function of time. Owing to spontaneous emission the Talbot effect is destroyed. We will see below that, for sufficiently large times, the Talbot periodicity is restored.

If both traveling wave components of the field are linearly-polarized along $\mathbf{z}$ and if the ground state angular momentum is equal to 0, then

$$N(\mathbf{n}) = {}^{3}/_{8\pi}\left(1-n_z^2\right). \tag{31}$$

We assume that the recoil frequency is sufficiently small to ensure that

$$\max\{1,\theta_1\}\,\omega_q/\Gamma \ll 1. \tag{32}$$

As a consequence one need only evaluate (28) at $\beta = 0$. Using the identity [33]

$$\int d\mathbf{n} n_i n_k \exp[i\mathbf{a}\cdot\mathbf{n}] = 4\pi a^{-3}\left\{(\sin a - a\,\cos a)\,\delta_{i,k} + \frac{a_i a_k}{a^2}\left[\left(a^2-3\right)\sin a + 3a\cos a\right]\right\} \tag{33}$$

one finds

$$C(\alpha,0) = {}^{3}/_{2}\alpha^{-3}\left[\alpha\cos\alpha + \left(\alpha^2-1\right)\sin\alpha\right]. \tag{34}$$

Note that $C(\alpha,0) \sim 1 - 3\alpha^2/16$ for $\alpha \ll 1$ and $C(\alpha,0) \sim \frac{3}{2\alpha}\sin\alpha$ for $\alpha \gg 1$.

The time scale of the transient response (25) is determined by the recoil frequencies,

$$\omega_d = 4\omega_k \sin(\Theta/2), \quad \omega_q = 4\omega_k \sin^2(\Theta/2) \tag{35a}$$

where Θ is the angle between the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$. There are essentially two time scales in the problem, one associated with spontaneous processes, $\tau_d = \omega_d^{-1}$, and one associated with stimulated processes $\tau_q = \omega_q^{-1}$. For Fourier components having $s \neq 0$, it is possible to isolate the contribution from stimulated processes since the contribution from spontaneous processes becomes negligible for $t \gg \tau_d$. It is not difficult to understand why spontaneous processes contribute a negligible amount in this limit. The recoil phase factor associated with spontaneous processes [34] is $e^{i\mathbf{k}_r\cdot\mathbf{q}t/2m} = e^{in_{r_x}\omega_d t}$. As mentioned above, this phase factor can be viewed as a recoil related Doppler phase. When

summed over all directions of the spontaneously emitted photon, it averages to zero for $\omega_d t \gg 1$. As a consequence, one finds from Eq. (25) that, for $\omega_d t \gg 1$ and $s \neq 0$,

$$\rho_{gg}(s,t) = {}^1/_2 \left\{ J_{2s}\left[\theta_1 \sin\left(\phi_{T_S}(st)/2\right)\right] + (-1)^s J_{2s}\left[\theta_1 \cos\left(\phi_{T_S}(st)/2\right)\right] \right\}. \tag{36}$$

In effect, Eq. (36) represents a periodic rephasing (Talbot effect) of *the ground state amplitude grating* that was created *immediately following* the radiation pulse, since for $\omega_d t \gg 1$ the spontaneous contribution no longer plays a role. One could have equally well ionized all the excited state atoms immediately following the pulse. The Fourier components, $\rho_{gg}(s,t)$, and total density, $\rho_{gg}(\mathbf{r},t) = \sum_s \rho_{gg}(s,t)\, e^{is\mathbf{q}\cdot\mathbf{r}}$, are periodic functions having period equal to $2\pi/\omega_q$ and can be used to measure recoil frequency [23,35], but, in contrast to scattering by phase gratings, there is no time for which the density is uniform when $\omega_d t \gg 1$. When all Fourier components are taken into account, and for large pulse areas (but not so large as to violate the Raman-Nath approximation),

$$(\omega_q \tau)^{-1} \gg \theta_1 \gg 1, \tag{37}$$

it can be shown that the atoms are focused by the field. This new regime of atom focusing, as well as its relation to focusing by phase gratings, will be considered in a future publication.

For earlier times, when $t \lesssim \omega_d^{-1} \lesssim \omega_q^{-1}$, the spontaneous term contributes to the atomic density. If $t \ll \omega_d^{-1}$, the total density is approximately uniform since spontaneous decay "refills" the "hole" in the ground state that is created by the radiation pulse. From Eqs. (25, 34), one finds that

$$\rho_{gg}(s,t) \approx \frac{1}{64}(s\omega_q t)^2 \theta_1^2 \delta_{s,1} + \frac{1}{10}(-1)^s (s\omega_d t)^2 J_{2s}(\theta_1). \tag{38}$$

The Fourier components build up as t^2 when $t \ll \omega_d^{-1}$. When the angle between wave vectors is small ($\Theta \ll 1$), and for somewhat larger times, $t \sim \omega_d^{-1} \ll \omega_q^{-1}$, Eqs. (25) reduces to

$$\rho_{gg}(s,t) = {}^1/_2 (-1)^s J_{2s}[\theta_1]\left[1 - C\left(\phi_{T_D}(st), 0\right)\right], \tag{39}$$

allowing one to isolate the contribution from spontaneous processes. The Fourier component $\rho_{gg}(1,t)$ is plotted in Fig. 1 as a function of $\omega_d t$; it is not a periodic function of $\omega_d t$. The time dependence of Eq. (39) can serve as a probe of the spontaneous emission distribution function [see Eq. (28)].

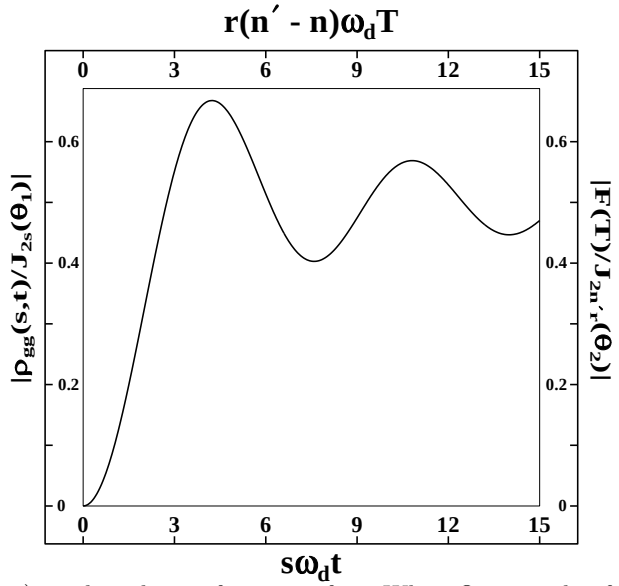

FIG. 1. The quantity $1 - C(x,0)$ is plotted as a function of x. When $\Theta \ll 1$, this function gives the time-dependence of the Fourier component $|\rho_{gg}(s,t)/J_{2s}(\theta_1)|$ $(x = s\omega_d t)$ for $t \sim \omega_d^{-1} \ll \omega_q^{-1}$ and $|F(T)/J_{2n'r}(\theta_2)|$ $[x = r(n'-n)\omega_d T]$ for $T \sim \omega_d^{-1} \ll \omega_q^{-1}$.

For counterpropagating waves ($\Theta = \pi$), the recoil frequencies ω_q and ω_d coincide and achieve their maximum value $\omega_d = \omega_q = 4\omega_k$. In this limit, atom interference effects occur on the shortest possible time scale. The Fourier component $\rho_{gg}(1,t)$ is plotted as a function of $\omega_q t$ in Fig. 2 for $\Theta = \pi$. Other Fourier components could be shown as

well, but $\rho_{gg}(1,t)$ is the component most easily monitored using backscattering techniques. One sees that, initially, $\rho_{gg}(1,t)$ is aperiodic, but it asymptotically approaches periodic behavior for $\omega_d t \gg 1$. The area $\theta_1 = 38.9$ is that for which $|\rho_{gg}(1,t)|$ achieves its maximum. The area $\theta_1 = 4.81$ is chosen to maximize the relative contribution of spontaneous processes. It was obtained by maximizing the ratio ρ_m/ρ_m^{as}, where ρ_m is the maximum of the exact expression (25), which occurs at $\omega_q t \sim 1$, and ρ_m^{as} is the maximum of the asymptotic expression (36) occurring at $\omega_q t \gg 1$.

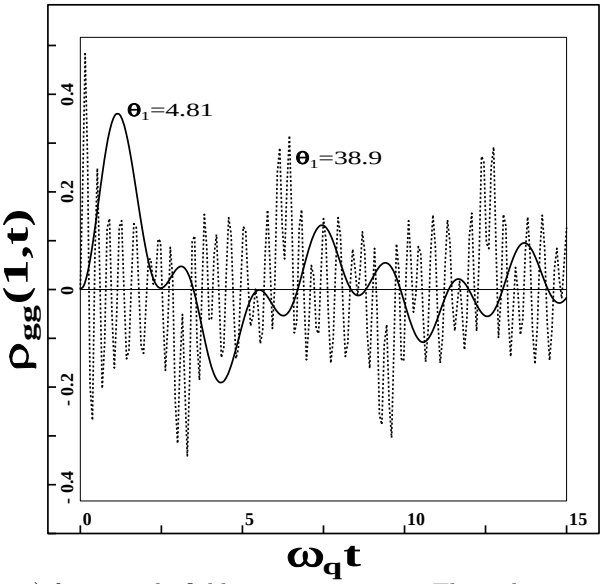

FIG. 2. Fourier component $\rho_{gg}(1,t)$ for a single field interaction zone. The pulse area $\theta_1 = 38.9$ (dashed curve) is chosen to maximize $\rho_{gg}(1,t)$, while the area $\theta_1 = 4.81$ (solid curve) is chosen to maximize the relative contribution from spontaneous processes.

IV. TWO FIELD INTERACTION ZONES

In the previous section, it was assumed that any Doppler dephasing was negligible on the time scales under consideration. In this section, we look at the limit in which

$$qu \gg \omega_d, \omega_q, \tag{40}$$

where u is the width of the velocity distribution in the direction of $\mathbf{q}$. To be specific, we take the momentum distribution as $W(\mathbf{p}) = W_\perp(\mathbf{p}_\perp)W_q(p_q)$,

$$W_q(p_q) = \frac{1}{\sqrt{\pi}mu} e^{-p_q^2/(mu)^2} \tag{41}$$

is the distribution of momenta in the direction of $\mathbf{q}$ and $W_\perp(\mathbf{P}_\perp)$ is the distribution of momenta transverse to $\mathbf{q}$. By combining Eqs. (25) and (41) one finds that, following a single pulse, the Fourier components of the density are given by

$$\rho_{gg}(s,t) = e^{-(squt)^2/4} J_{2s}\left[\theta_1 \phi_{T_S}(st)/2\right], \tag{42}$$

correct to order ω_d/qu. This term survives only for times of order $(squ)^{-1}$, which implies that the argument of the Bessel function is of order $\theta_1\omega_q/qu$. This, in turn, implies that only Fourier components having $s \sim \max\{1, \theta_1\omega_q/qu\}$ contribute significantly for times in which Fourier components other than $s = 0$ are nonvanishing.

The picture is rather simple. The radiation pulse excites the atoms which then decay back to the ground state giving a uniform density. For times $t < (squ)^{-1}$, the sth Fourier component begins to build up significantly provided $\theta_1\omega_q/qu > s$, and a modulated atomic density appears. For times $t > (qu)^{-1}$, all spatial modulation has been washed out as a result of Doppler dephasing. Since this time is shorter than the inverse recoil frequencies, the main features of the time dependence $\rho_{gg}(s,t)$ found in Sec. III for homogeneous broadening never can be realized in this inhomogeneously broadened sample.

The spatial modulation is not lost, however, and can be restored using echo techniques if a second pulse is applied at some time T following the initial pulse. The time dependence of $\rho_{gg}(s,t,T)$, considered as a function of T, displays the same features found in the previous section for $\rho_{gg}(s,t)$ as a function of t. Specifically, it can be used as a probe of spontaneous processes.

In this section, we consider the atomic response following two pulses centered at $t = 0$ and $t = T$ ($T_1 = 0$ and $T_2 = T$). One key point to remember is that the relevant time window for which the modulation is restored is of order $(squ)^{-1}$. Thus, it is possible that the Doppler dephasing associated with spontaneous emission no longer plays a critical role in killing off the Fourier components, since this dephasing is negligible on a time scale $(squ)^{-1}$. We shall see this to be the case; as a consequence the Fourier components at the "echo times" have contributions from *both* the stimulated and spontaneous terms, even for $\omega_d t \gg 1$.

The Fourier components of the density at times $t - T_2 \gg \Gamma^{-1}$ following the second pulse can be written as

$$\rho_{gg}(s,\mathbf{p},t) = \rho_{gg}^{(SS)}(s,\mathbf{p},t) + \rho_{gg}^{(DS)}(s,\mathbf{p},t) + \rho_{gg}^{(SD)}(s,\mathbf{p},t) + \rho_{gg}^{(DD)}(s,\mathbf{p},t), \tag{43}$$

where $\rho_{gg}^{(IK)}(s,\mathbf{p},t)$ represents the contribution from stimulated ($K = S$) or spontaneous ($K = D$) processes following the second pulse that depend on the stimulated ($I = S$) or spontaneous ($I = D$) component of the ground state density matrix Fourier components that were created by the first pulse. Consider, for example $\rho_{gg}^{(SS)}(s,\mathbf{p},t)$. Using Eqs. (11a, 17, 22, 41), one finds that the Fourier component

$$\rho_{gg}^{(SS)}(s,t) = \int d\mathbf{p}\rho_{gg}^{(SS)}(s,\mathbf{p},t), \tag{44}$$

is given by

$$\begin{aligned}\rho_{gg}^{(SS)}(s,t) = &\sum_{s'} \exp\left\{-n^2q^2u^2\left[s(t-T)+(s-s')T\right]^2/4\right\} \\ &\times \sum_{\ell_1} \exp\left\{-2i\omega_q\left[\ell_1-(s-s')/2\right]\left[s(t-T)+(s-s')T\right]\right\} J_{2\ell_1}(\theta_1/2)\, J_{2(\ell_1-s+s')}(\theta_1/2) \\ &\times \sum_{\ell_2} \exp\left[-2is\omega_q(t-T)(\ell_2-s'/2)\right] J_{2\ell_2}(\theta_2/2)\, J_{2(\ell_1-s')}(\theta_2/2)\end{aligned} \tag{45}$$

When the time separation between pulses is larger than the inverse Doppler width,

$$quT \gg 1, \tag{46}$$

the average over momenta leads to a non-vanishing contribution only if

$$s(t-T) + (s-s')T \lesssim (qu)^{-1}. \tag{47}$$

Inequality (47) can be satisfied in the vicinity of the echo times, t_e, defined as

$$t_e = \frac{n'}{n}T, \tag{48}$$

where n' and n ($n' > n$) are positive integers having no common factors, provided that

$$s = nr,\ s' = n'r, \tag{49}$$

where r is an integer. Setting

$$\delta t = t - t_e = t - \frac{n'}{n}T,$$

and using inequality (47) and Eqs. (49), one finds that the Doppler phase is nondestructive for times

$$\delta t \lesssim 1/(nrqu)$$

Although not indicated explicitly, δt is a function of t, n, n' and T.

Since $nr = s$, Eq. (49) implies that the (nr)th Fourier component ($r = 1, 2, 3\ldots$) is nonvanishing in the vicinity of the echo time. For example, if $n = 1$, all the Fourier components contribute near the echo times $t = n'T$ ($n' = 2, 3\ldots$),

corresponding to a macroscopic atomic grating having period $\lambda/\left[2\sin\left(\Theta/2\right)\right]$; if $n=2$, the $(2r)$th Fourier components contribute near the echo times $t=n'T/2$ $(n'=3,5,7\ldots)$, corresponding to a macroscopic atomic grating having period $\lambda/\left[4\sin\left(\Theta/2\right)\right]$. In this manner one can generate macroscopic atomic gratings having period $\lambda/\left[2n\sin\left(\Theta/2\right)\right]$. Note that condition (47) for a nondestructive Doppler phase is a *classical* condition since it does not contain $\hbar$. The shape of the echo signal about the echo times and the dependence of the signal on the time separation of the pulses is determined by effects related to quantum scattering, but the actual *location* of the signals is determined by classical considerations only [8].

Using Eq. (23a) for the sums over ℓ_1 and ℓ_2, one finds that, at $\delta t=t-(n'/n)T$,

$$\begin{aligned}\rho_{gg}^{(SS)}\left(nr,\delta t\right) &= {}^{1}\!/_{4}\exp\left[-(nrqu\delta t)^{2}/4\right]\\ &\times\left\{J_{2r(n'-n)}\left[\theta_{1}\sin\left(\phi_{T_S}\left(nr\delta t\right)/2\right)\right]+(-1)^{(n'-n)r}J_{2r(n'-n)}\left[\theta_{1}\cos\left(\phi_{T_S}\left(nr\delta t\right)/2\right)\right]\right\}\\ &\times\left\{J_{2rn'}\left[\theta_{2}\sin\left(r\phi_{TL_S}/2\right)\right]+(-1)^{n'r}J_{2rn'}\left[\theta_{2}\cos\left(r\phi_{TL}/2\right)\right]\right\},\end{aligned}\tag{50}$$

where ϕ_{TL_S} is a Talbot-Lau phase, defined as

$$\phi_{TL_S}=\left(n'-n\right)\omega_{q}T.\tag{51}$$

It is easy to show that the Fourier component $\rho_{gg}^{(SS)}\left(nr,\delta t=0\right)$ does not vanish for $s\neq 0$. We shall return to this point shortly.

The remaining terms in Eq. (43) can be evaluated in the same manner, and one finds

$$\rho_{gg}\left(nr,\delta t,T\right)=\Phi\left(\delta t\right)F\left(T\right),\tag{52a}$$

$$\begin{aligned}\Phi\left(\delta t\right) &= {}^{1}\!/_{2}\exp\left[-(nrqu\delta t)^{2}/4\right]\left\{J_{2r(n'-n)}\left[\theta_{1}\sin\left(\phi_{T_S}\left(nr\delta t\right)/2\right)\right]\left[1+C\left(\phi_{T_D}\left(nr\delta t\right),-r\left(n'-n\right)\omega_{d}/\Gamma\right)\right]\right.\\ &\left.+(-1)^{r(n'-n)}J_{2r(n'-n)}\left[\theta_{1}\cos\left(\phi_{T_S}\left(nr\delta t\right)/2\right)\right]\left[1-C\left(\phi_{T_D}\left(nr\delta t\right),-r\left(n'-n\right)\omega_{d}/\Gamma\right)\right]\right\},\end{aligned}\tag{52b}$$

$$\begin{aligned}F\left(T\right) &= {}^{1}\!/_{2}\left\{J_{2rn'}\left[\theta_{2}\sin\left(r\phi_{TL_S}/2\right)\right]\left[1+C\left(r\phi_{TL_D},rn\omega_{q}/\Gamma\right)\right]\right.\\ &\left.+(-1)^{rn'}J_{2rn'}\left[\theta_{2}\cos\left(r\phi_{TL_S}/2\right)\right]\left[1-C\left(r\phi_{TL_D},rn\omega_{q}/\Gamma\right)\right]\right\},\end{aligned}\tag{52c}$$

where

$$\phi_{TL_D}=\left(n'-n\right)\omega_{d}T.\tag{53}$$

The result (52a) is the product of a term, $\Phi\left(\delta t\right)$, giving the time dependence of the Fourier component in the vicinity of the echo time, and a term, $F\left(T\right)$, giving its dependence on the time separation of the pulses.

We consider these terms separately, starting with $\Phi\left(\delta t\right)$. The exponential factor in Eq. (52b) leads to a nonvanishing contribution to the atomic density only for times $nr\delta t\sim(qu)^{-1}\ll\omega_{d}^{-1}<\omega_{q}^{-1}$. In this limit, and for $\omega_{d}/\Gamma\ll 1$, Eq. (52b) reduces to

$$\Phi\left(\delta t\right)=\exp\left[-(nrqu\delta t)^{2}/4\right]J_{2r(n'-n)}\left[\theta_{1}\phi_{T_S}\left(nr\delta t\right)/2\right].\tag{54}$$

Comparing this expression with Eq. (42), one sees that the atomic density near the echo times mirrors the atomic density in the time interval δt following the first pulse. Thus the dependence of the Fourier component $s=nr$ near the echo time can be understood in terms of the dependence of the sth Fourier component at a time δt following the first pulse. In this time interval, only those Fourier components having $s\lesssim\theta_{1}\omega_{q}/qu$ are created with nonnegligible amplitude.

It is important to note that the recoil dephasing responsible for the "washing out" of the spontaneous contribution in the Talbot effect plays no role here, since it is negligibly small in the time interval $\delta t\sim(qu)^{-1}$. The recoil dephasing *during* the spontaneous decay of the excited state *does* provide a small correction to Eq. (54) for $\delta t=0$. It follows from Eq. (52b) that, to lowest order in $\omega_{d}/\Gamma\ll 1$

$$\Phi\left(\delta t=0\right)\approx{}^{1}\!/_{5}\left(-1\right)^{r(n'-n)}\left[r\left(n'-n\right)\omega_{d}/\Gamma\right]^{2}J_{2r(n'-n)}\left(\theta_{1}\right).\tag{55}$$

We return now to the dependence of the Fourier components on T, given by $F\left(T\right)$. The echo configuration considered in this section is the same as that which leads to the Talbot-Lau effect. In the (matter wave) Talbot-Lau effect, the atomic density is a periodic function of the time separation between the pulses. In contrast to the normal Talbot-Lau effect, the density (52a) is *not* a periodic function of T, owing to the spontaneous contributions to $F\left(T\right)$. However,

for pulse separations $T \gg \omega_d^{-1}$, the spontaneous processes no longer contribute to $F(T)$ and one finds the periodic dependence

$$F(T) \approx {}^1/_2 \left\{ J_{2rn'} [\theta_2 \sin(r\phi_{TL_S}/2)] + (-1)^{rn'} J_{2rn'} [\theta_2 \cos(r\phi_{TL_S}/2)] \right\}, \tag{56}$$

which is reminiscent of Eq. (36). For shorter pulse separations, $T \ll \omega_d^{-1}$, the function $F(T)$ builds up as

$$F(T) \approx \frac{1}{10} (-1)^{rn'} J_{2rn'}(\theta_2) [r(n'-n)\omega_d T]^2 . \tag{57}$$

[compare with Eq. (38)]. When the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$ is small, one finds that, for $T \sim \omega_d^{-1} \ll \omega_q^{-1}$,

$$F(T) = {}^1/_2 (-1)^{n'r} J_{2n'r} [\theta_2] [1 - C(r\phi_{TL_D}, 0)] \tag{58}$$

which is to be compared with Eq. (39). The Talbot-Lau dependence $F(T)$ [See Eq. (52c)] is qualitatively similar to the Talbot dependence of $\rho_{gg}(s,t)$ [See Eq. (25)]. In the limit that $\Theta \ll 1$, and for $T \sim \omega_d^{-1}$ [Eq. (58)] or $t \sim \omega_d^{-1}$ [Eq. (39)], the agreement is quantitative.

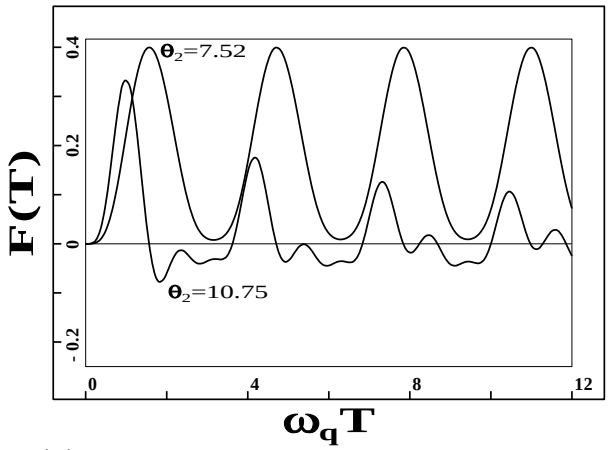

FIG. 3. A graph of the function $F(T)$ that gives the dependence of the Fourier component $\rho_{gg}(1, \delta t, T)$ on the separation between pulses. The pulse area $\theta_2 = 7.52$ is chosen to maximize $F(T)$, while the area $\theta_2 = 10.75$ is chosen to maximize the relative contribution from spontaneous processes.

For the case of counterpropagating waves ($\Theta = \pi$), the dependence $F(T)$ is plotted in Fig. 3 for $n = 1$, $r = 1$ ($s = nr = 1$) and $n' = 2$. The pulse area $\theta_2 = 7.52$ was chosen to maximize $F(T)$, while the area $\theta_2 = 10.75$ was chosen to maximize the contribution from spontaneous processes. The value $\theta_2 = 10.75$ was obtained by maximizing the ratio F_m/F_m^{as}, where F_m is the maximum of the exact expression (52c), which occurs at $\omega_q T \sim 1$, and F_m^{as} is the maximum of the asymptotic expression (56) occurring at $\omega_q T \gg 1$.

V. SUMMARY

We have described a new type of matter wave atom interferometer (MWAI). One or two standing wave, resonant pulses interact with an ensemble of atoms. Atomic motion during the pulse is neglected (Raman-Nath approximation), as is spontaneous emission. As a result of the atom-field interactions, the total atomic density acquires a spatial modulation that can be attributable solely to matter-wave interference - the signals arise only for times greater than or comparable with the inverse recoil frequency ω_d^{-1} or ω_q^{-1}. Spontaneous processes destroy the periodicity of the Talbot or Talbot-Lau signals. However, for sufficiently long times, the spatially modulated atomic density becomes a periodic function of $\omega_q t$ (Talbot effect) or $\omega_q T$ (Talbot-Lau effect).

To observe the Talbot effect discussed in Sec. III, one can use a highly collimated ($qu\theta_b \ll \omega_q$) atomic beam that is sent through a field interaction region. The fields can be pulsed, if necessary, to ensure that the interaction time is much less than the excited state lifetime. The modulated atomic density can be monitored by scattering a probe field off the atoms or by directly depositing the atoms on a substrate. One might also contemplate doing this experiment in the time domain, using a Bose condensate. The Talbot-Lau effect can be observed either in the spatial domain

(using an atomic beam having an appropriate angular divergence) or in the time domain, using a laser cooled and trapped vapor.

Finally, we would like to comment on the fact that the Talbot-Lau Fourier components do not vanish identically for $\delta t = 0$ [see Eq. (55)]. The amplitude of these components is of order $(\omega_d/\Gamma)^2$, reflecting the contribution of recoil dephasing on the time scale of the excited state lifetime. Although we did not give the equation in the text, there is also a contribution to the Talbot Fourier components of order $(\omega_d/\Gamma)^2$. These contributions reflect the "opening" of the closed two-level system by the recoil associated with spontaneous emission. As in the recoil-induced resonances [36], the opening of the system is connected with quantum scattering - it vanishes in the limit that $\hbar \sim 0$.

This situation differs from that involving phase gratings on *open*, two-level transitions. Imagine that the atoms have two ground states, g and g', to which the excited state can decay, but that the field drives only the $g-e$ transition. By using a far detuned field, spontaneous emission to state g' can be suppressed by a factor $(\Gamma/\Delta)^2$. Following decay, but for times much less than the inverse recoil time, the population density ρ_{gg} is spatially modulated to order $(\Gamma/\Delta)^2$, as is $\rho_{g'g'}$, but the total density $(\rho_{gg} + \rho_{g'g'})$ is uniform. The opening of $e-g$, two-level system in this case has nothing to do with quantum scattering effects.

ACKNOWLEDGMENTS

We are pleased to acknowledge helpful discussions with J. L. Cohen and M. Weitz. This research is supported by the U. S. Army Research office under grant number DAAG5-97-0113 and by the National Science Foundation under grants PHY-9414020 and PHY-9800981.

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